Piezoelectric composite lattices with high sensitivity

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Abstract

Lattice structures are presented with a piezoelectric sensitivity, d, much larger in magnitude than that of material comprising the lattice ribs. Large sensitivity is achieved by using piezoelectric bimorph elements as ribs; these bend in response to electric input, giving rise to a much larger displacement than for axial elements. Complex electrical connectivity is helpful but not necessary; surface contact can be sufficient. Lattices are amenable to piezoelectric vibration damping. Piezoelectric sensitivity can be arbitrarily large and is unbounded.

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1 Introduction

1.1 Piezoelectric materials

Piezoelectric materials produce an electric polarization when stressed mechanically and deform in response to an electric field. They are always anisotropic. Specifically [1], the strain ϵ_{ij}, depends upon stress σ_{kl} via the elastic compliance J_{ijkl}, upon electric field $\mathcal{E}_k$ in piezoelectric materials with modulus tensor d_{kij} at constant temperature, and on temperature change ΔT (via the thermal expansion α_{ij}). Moreover the electric displacement vector $\mathcal{D}_i$ depends on electric field via K_{ij} which is the dielectric tensor at constant stress and temperature, and it can depend on temperature change ΔT via the pyroelectric effect; p_i is the pyroelectric coefficient at constant stress:

$$\epsilon_{ij} = J_{ijkl}\sigma_{kl} + d_{kij}\mathcal{E}_k + \alpha_{ij}\Delta T \tag{1}$$

$$\mathcal{D}_i = d_{ijk}\sigma_{jk} + K_{ij}\mathcal{E}_j + p_i\Delta T \tag{2}$$

If there are phase angles, material properties associated with response to sinusoidal input in time become complex quantities in the context of viscoelasticity and dielectric relaxation [2]. Properties are expressed in the reduced notation in which the two indices associated with stress or strain are reduced to one, so $d_{333} \to d_{33}$. The most sensitive piezoelectric materials are ceramics for which the sensitivity of charge to force d_{33} is from 100 to 600 pC/N, in comparison with 10 to 20 pC/N, for piezoelectric polymers. The sensitivity of displacement to input voltage has the same value by virtue of reciprocity, provided the material is elastic; the units pm/volt are equivalent to pC/N.

Piezoelectric composite materials have been conceptualized with particulate, fibrous, platelet microstructure or they may have a more complex geometry. A set of rule-of-mixtures type equations [3] was provided to approximate the effective piezoelectric coefficients of piezoelectric composites of simple structure. For d_{31} it is a Voigt equation; for d_{33} the piezoelectric coefficients are weighted by the dielectric constant as well as the volume fraction. Fibrous piezoelectric composites have also been analyzed [4]. Piezoelectric composites have advantages over homogeneous piezoelectric ceramic materials. For example, they may offer increased electromechanical coupling efficiency, also a decrease in specific acoustic impedance, and suppression of unwanted modes of vibration [5]. Too, the voltage coefficient g_{33} (open circuit electric field divided by stress) of a laminar composite of piezoelectric and compliant dielectric layers can substantially exceed that of the piezoelectric constituent alone [3].

1.2 Bounds on properties

Bounds on various physical properties have been developed [6]. For example, the upper and lower bounds of *stiffness* of two phase and multi-phase composites have been obtained by Hashin [7], and by Hashin and Shtrikman [8]. Further analysis on composites is reviewed by Hashin [9] and Milton [6]. Bounds on thermoelastic properties of two phase composites [10] are weighted averages of the expansion values of the constituents. Bounds similar to elastic Voigt and Reuss bounds and to Hashin-Shtrikman bounds have been determined for piezoelectric composites [11] [12].

Arbitrarily high positive or negative thermal expansions can be achieved in lattice type composites with void content [13] [14] or in dense composites with interfaces that allow slip. Composites in which the 'inclusions' are voids or cells, filled with air or liquid, are called cellular solids. Lattices, including two-dimensional honeycombs, are cellular solids in which the cells have a regular repeating structure. Zero expansion can be attained in such lattices [15]. These composites contain rib elements that themselves have composite microstructure, corresponding to a low level of structural hierarchy [16]. Each rib element is a bi-layer made of two bonded layers of differing thermal expansion coefficient. Such materials can exceed the limits imposed by bounds that tacitly assume there is no void space and no slip between constituents.

In the present research, we develop piezoelectric lattices based on composite rib elements. We find that piezoelectric sensitivity d can be arbitrarily greater than that of material in the ribs.

2 Lattice structures and properties

2.1 Lattice sensitivity analysis

Several triangular piezoelectric lattices are shown in Figure 1. Rib elements that undergo bending in response to an electrical signal are chosen as piezoelectric elements. Each rib of length L and full thickness h in the lattice consists of two layers of piezoelectric material, arranged as a bimorph, a kind of bender element. When a voltage is applied to such a rib, one layer expands, and the other contracts, giving rise to bending. In the figures, dark and light shading of the the ribs denotes different polarization directions of layers in the bimorph ribs. Some electrical connections from +, - electrodes are indicated schematically by thin lines. Commercial bimorph strips in our laboratory have electrodes near the ends, i. e. the shortest sides; the remaining surfaces are covered with an insulating layer; end connection is not shown in the figures to avoid clutter with the end contact.

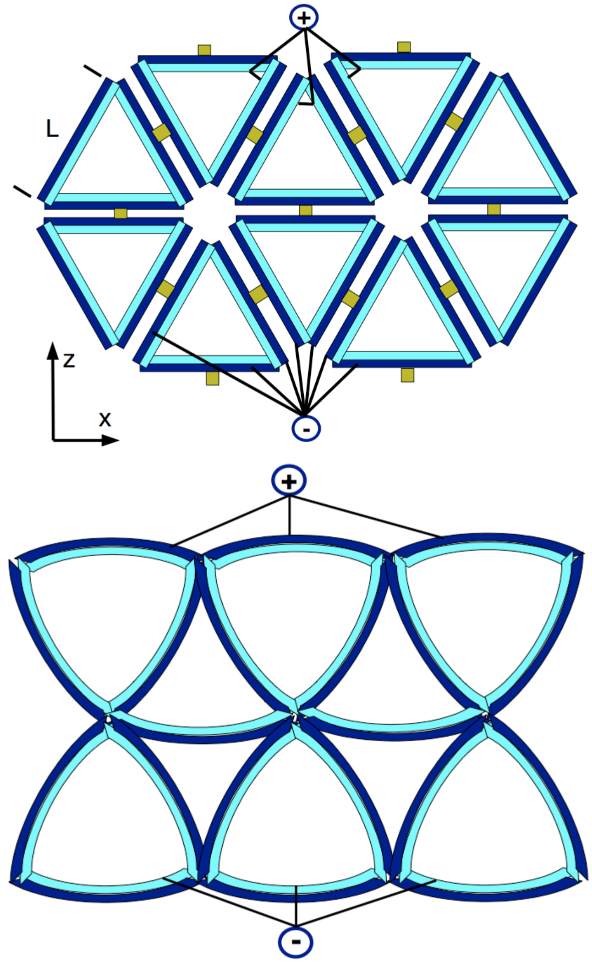
Figure 1(a) shows a fully bend dominated lattice in which each rib is free to bend in response to an electrical signal. A fully triangulated lattice with curved ribs shown in Figure 1(b) approaches stretch dominated behavior if the curvature is small. Specifically the elastic (Young's) modulus E normalized to the modulus E_s of the solid comprising the ribs, is $E/E_s \propto [\frac{h}{L}]$. This goes as the inverse first power of the rib slenderness corresponding to greater stiffness than bend dominated structures, as shown for a bi-layer thermoelastic lattice [15]. Rib curvature is needed to achieve thermoelastic or piezoelectric response: change in curvature results in overall dimensional changes in the lattice. Ribs of sufficient curvature give rise to bend dominated behavior $E/E_s \propto [\frac{h}{L}]^3$. Straight ribs are of interest in that available bimorph elements can be used, therefore such lattices are analyzed.

For the edge contact lattice in Figure 1(a), the ribs are free to bend unless the lattice is subject to a constraint. Stiff spacers (small rectangles) transmit the bending motion among adjacent cells. If the spacers are electrically conductive, the outer surfaces, if conductive, of the cells are at a common potential that can be imposed by a voltage V applied at the surface, but wiring is still needed to impose a difference in potential on the inner surfaces. The deflection per volt for a single bimorph cantilever rib of length L and full thickness h containing two anti-parallel piezoelectric layers of sensitivity d_{31} and thickness $h/2$ is [17] $(u/V)_{rib} = -\frac{3}{2}d_{31}[\frac{L}{h}]^2$. Elastic structural compliance (displacement / force) is $u/F = \frac{2}{w}S_{11}[\frac{L}{h}]^3$ with w as the width and S_{11} as the material compliance, inverse of Young's modulus E. A bender in which one layer is piezoelectric and the other layer is not piezoelectric, is called a unimorph; the analysis is more complicated [18] than for a bimorph.

The lattice of bimorph ribs contains multiple cells of angle θ between ribs: n cells thick in the z direction, m cells thick in the x direction. Each of the n layers of cells has two horizontal ribs that contribute fully $\frac{1}{4}(u/V)_{rib}$, because the center of a rib moves 1/4 as much as the end of a cantilever. Each layer also has two oblique ribs that contribute $\frac{1}{4}(u/V)_{rib}sin(\frac{\theta}{2})$.

So the sensitivity is as follows:

$$d_{33}^{eff} = -d_{31}\frac{3}{2}\frac{2n}{4}(1 + sin(\frac{\theta}{2}))[\frac{L}{h}]^2. \quad (3)$$



(a) (b)

Figure 1: Triangular piezoelectric lattices. (a), fully bend-dominated lattice. (b), lattice with curved ribs.

The sign of d_{33}^{eff} depends on the rib orientation; interchange of light and dark layers results in a reversal of sign. This sensitivity is much larger in magnitude than the intrinsic piezoelectric coefficient of the material from which the ribs are made, owing to the enhancement of displacement by bending. For the lattice segment shown, $n = 2$ and $\theta = 60°$. Slender ribs give rise to enhanced piezoelectric sensitivity. The in-plane elastic modulus E of this lattice normalized to the modulus of the solid rib material is, neglecting the spacer compliance, $E/E_s = \frac{1}{2}\frac{cos(\frac{\theta}{2})}{1+sin(\frac{\theta}{2})}[\frac{h}{L}]^3$. The transverse sensitivity has contributions from m cells in the horizontal direction. In each cell there two ribs that contribute $\frac{1}{4}(u/V)_{rib}cos(\frac{\theta}{2})$. So $d_{31}^{eff} = d_{31}\frac{3}{2}\frac{2m}{4}cos(\frac{\theta}{2})[\frac{L}{h}]^2$.

A piezoelectric lattice based on two dimensional chiral honeycomb [19] is shown in Figure 2. This was originally devised for its elastic properties: a negative Poisson's ratio that tends to the lower bound -1, independent of strain [19] as ribs become more slender; slender beam theory is used throughout. For the corresponding piezoelectric lattice, reversal in bimorph orientation is incorporated in each rib to provide for the appropriate deformation shape of the rib, a sigmoid curve. Electrical insulation must therefore be provided at the rib midpoints. The elastic modulus of this type of chiral honeycomb is $E/E_s = \sqrt{3}[\frac{h}{L_c}]^3[\frac{L_c}{r}]^2$, with r as the node radius, L_c as the

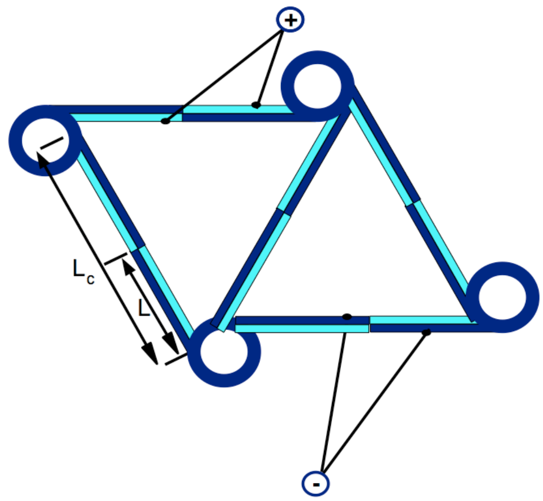


Figure 2: Chiral piezoelectric lattice.

rib length and E_s as the solid rib modulus. Deformation is bend-dominated; the circular nodes impose bending moments on the ribs as the lattice is compressed or stretched. Elastic properties are isotropic in-plane provided the ribs are of equal length so the angle between them is $\theta = 60°$ as shown.

The in-plane Poisson's ratio approaches -1 for slender ribs. The displacement u of the rib-node attachment point of a cell in terms of the bend angle is $u = r\phi$; bend angle is between the deformed rib and its initial slope at its attachment to the node. Bend angle can result from moments from external forces [19] or from electrical input. For the latter case, the angle per volt for a single bimorph rib of length L is $\phi/V = -3d_{31}[\frac{L}{h^2}]$ [17]. The strain is $\epsilon = \phi r/R$ with $R = \sqrt{(\frac{L_c}{r})^2 + 4}$ as the distance between node centers. The lattice sensitivity is $d_{33}^{eff} = \frac{u_y}{V} = \frac{1}{V} n\epsilon L_c cos(\frac{\theta}{2})$ because n cells are assumed in the y direction and the projection of length in the y direction is taken. The effective piezoelectric sensitivity of the lattice is as follows with $L_c = 2L$:

$$d_{33}^{eff} = -d_{31}\frac{3}{2}n\frac{cos(\frac{\theta}{2})}{\sqrt{(\frac{L_c}{r})^2 + 4}}[\frac{L_c}{h}]^2 \tag{4}$$

The piezoelectric sensitivity d increases as the square of the rib slenderness; the elastic modulus decreases as the cube of rib slenderness as with the edge contact lattice in Figure 1(a). Chiral lattices are bend dominated yet they allow an independent control over stiffness and piezoelectric sensitivity via the node radius r. As r becomes small in comparison with rib length, the elastic modulus increases as the inverse square of r; the piezoelectric sensitivity decreases linearly with r. The axial strain in all directions is equal because this structure easily accommodates area change but resists shape change, so if the dimension in each direction is the same, $d_{31}^{eff} = d_{33}^{eff}$.

2.2 Continuum view of lattices

2.2.1 Equivalent continuum and applications

As for the continuum view and scaling, there is no length scale in the classical theory of elasticity or piezoelectricity, so lattices may be of any size, from macroscopic to nano-scale. The above

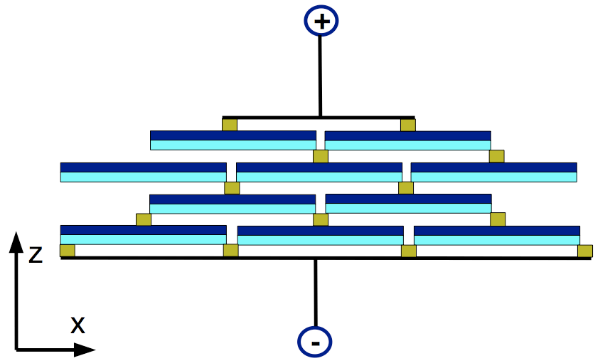


Figure 3: Piezoelectric lattice that allows a voltage to be applied at the surface. Electrically conductive spacers (small rectangles) provide an electrical and mechanical path through the lattice.

piezoelectric lattices differ from elastic or thermoelastic lattices in that they should not be viewed as equivalent continua. Elastic lattices or foams deform in response to stress at the surface, and thermoelastic lattices deform in response to overall temperature changes. The above piezoelectric lattices, by contrast, receive electrical input via connectivity to appropriate ribs. That results in a dependence of effective sensitivity on the number of cells in a particular direction, in contrast to elastic and thermoelastic lattices. Similarly, stacks of axial piezoelectric elements have electrical connectivity such that a low voltage applied results in a sum of displacements of each element. If an equivalent continuum amenable to surface electrode contact is desired, a structure such as that in Figure 3 may be considered. The spacers here can be electrically conductive or they may have sufficiently high dielectric constant to provide a much lower impedance than the piezoelectric elements. Suppose there are n layers in the z direction. The overall displacement is the sum of the displacement of each layer; the imposed voltage is shared among the layers so there is no dependence of sensitivity on cell number. The sensitivity is $d_{33}^{eff} = -d_{31}\frac{3}{2}[\frac{L}{h}]^2$. Sensitivity is much greater than that of an axial stack, due to bending displacement. All the motion is in the z direction so $d_{31}^{eff} = 0$ in contrast to the prior lattices. The void content of this structure, in contrast to the prior ones, is controllable independently of the rib slenderness, by varying the vertical thickness of the spacers. Void content can be made arbitrarily small.

Macroscopic piezoelectric lattices may be assembled from commercial bimorph or unimorph bender elements; for macroscopic or microscopic lattices, prototype methods previously used for composites [20] are applicable. The lattices presented here are distinct from piezoelectric superlattices [21] which are layered structures on a fine scale that exhibit dispersion of waves [22].

Applications are envisaged in the context of actuators, energy harvesting, and structures that change shape in response to stimuli. Airplane wings that change shape have been considered in the context of chiral honeycomb [23]. Vibration damping in structures is enabled by viscoelastic damping that occurs in the piezoelectric material and interface layers, and this may be incorporated into lattices via the concept of Forward [24]. Specifically, appropriately matched electrical resistance connected to the circuit provides mechanical damping via the electromechanical coupling that is inherent in piezoelectric materials. Such damping may be useful in satellites and other aerospace applications in which vibration damping is beneficial. The captured energy may also be harvested.

2.2.2 Two-phase bounds on effective properties and how to exceed them

The piezoelectric sensitivity is unbounded for these lattices; arbitrarily large values can be attained via sufficiently slender ribs. Thin ribs give rise to reduced stiffness. Too, these lattices are bend dominated so they are much more compliant than stretch-dominated structures that are optimized for stiffness. As in the case of thermal expansion, mathematical bounds for two phase composites do not apply to lattices because the bound analysis tacitly assumes the two phases are perfectly attached together with no slip or void space. Such an assumption, often not stated explicitly, does not apply to the lattice structures considered here.

The lattice concept can be readily extended to three dimensions. For example, cubical cells can be envisaged with spacers similar to those used to separate triangles in Figure 1(a).

The chiral lattice allows some recovery of stiffness if nodes of small size are used. In the limit of zero node size, the lattice becomes fully triangulated and stretch dominated, with no piezoelectric sensitivity; the bending analysis does not apply to that case.

3 Conclusions

To conclude, piezoelectric lattices based on bimorph ribs can have a piezoelectric d sensitivity much larger in magnitude than that of material comprising the lattice ribs. Piezoelectric sensitivity is unbounded. High sensitivity is achieved using ribs that bend in response to an input voltage. Chiral lattices with Poisson's ratio approching -1 in plane provide additional control over the sensitivity and modulus.

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